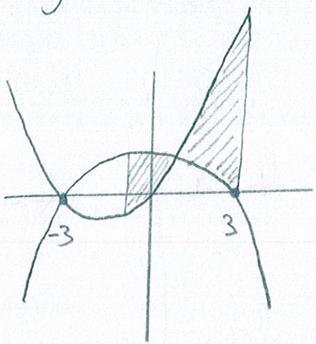
Find the area between the curves $y = 36 - 4x^2$ and $y = 2x^2 + 6x$ over the interval $[-1, 3]$. **SCORE: _____ / 8 PTS**

NOTE: Your final answer must be a number, not an integral nor sum of integrals.

$y = 36 - 4x^2 \rightarrow x\text{-INT} = \pm 3$

$y = 2x^2 + 6x \rightarrow x\text{-INT} = 0, -3$



$1\frac{1}{2}$ $\left[\int_{-1}^{2} \underbrace{(36 - 4x^2 - (2x^2 + 6x))}_{1} dx\right] + 1\frac{1}{2}\left[\int_{2}^{3} \underbrace{(2x^2 + 6x - (36 - 4x^2))}_{1} dx\right]$

$= \int_{-1}^{2} (36 - 6x - 6x^2)\, dx + \int_{2}^{3} (6x^2 + 6x - 36)\, dx$

$= \underbrace{(36x - 3x^2 - 2x^3)}_{1}\Big|_{-1}^{2} + \underbrace{(2x^3 + 3x^2 - 36x)}_{1}\Big|_{2}^{3}$

$= 36(2 - -1) - 3(4 - 1) - 2(8 - -1)$

$+ 2(27 - 8) + 3(9 - 4) - 36(3 - 2)$

$= 108 - 9 - \cancel{18} + \cancel{38} + \cancel{15} - 36$

$-3 \quad 2$

$= \underbrace{98}_{1}$

$36 - 4x^2 = 2x^2 + 6x$

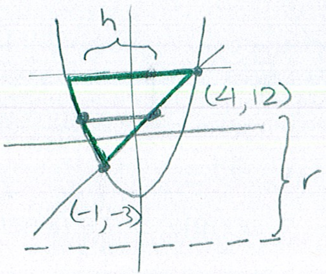
$0 = 6x^2 + 6x - 36$

$0 = 6(x^2 + x - 6)$

$0 = 6(x + 3)(x - 2)$

$x = -3, 2$

The region defined by $y \geq x^2 - 4$, $y \geq 3x$ and $y \leq 12$ is revolved around the line $y = -8$. **SCORE: _____ / 8 PTS**
Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid **using as few integrals as possible**.



$$\underbrace{2\pi}_{1} \overbrace{\int_{-3}^{12}}^{2} \underbrace{(y - -8)}_{2} \underbrace{\left(\tfrac{1}{3}y - -\sqrt{y+4}\right)}_{3} dy$$

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1, 4$

$\downarrow \quad \downarrow$

$y = -3, 12$

TALK TO ME IF YOU WROTE
dx WASHER METHOD INTEGRALS

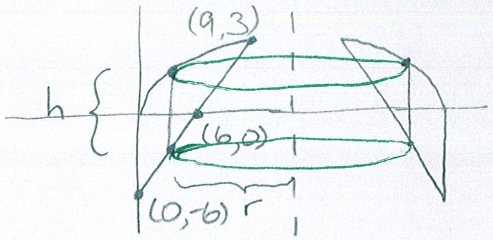
$y = x^2 - 4 \longrightarrow x = \pm\sqrt{y+4}$ $\quad x = -\sqrt{y+4}$ FOR LEFT SIDE OF PARABOLA

$y = 3x \longrightarrow x = \frac{1}{3}y$

Consider the region bounded by $y=\sqrt{x}$, $y=x-6$ and $x=0$. **SCORE: _____ / 14 PTS**

[a] If the region is revolved around the line $x=12$,
write, **<u>BUT DO NOT EVALUATE</u>**, an integral (or sum of integrals) for the volume of the solid

[i] using a dx integral **<u>(NOTE: You do NOT need to simplify your integrand.)</u>**



$$\underbrace{2\pi}_{1}\underbrace{\int_0^9}_{1} \underbrace{(12-x)}_{1}\underbrace{(\sqrt{x}-(x-6))}_{1}\,dx$$

$\sqrt{x} = x-6$

$x = x^2-12x+36$

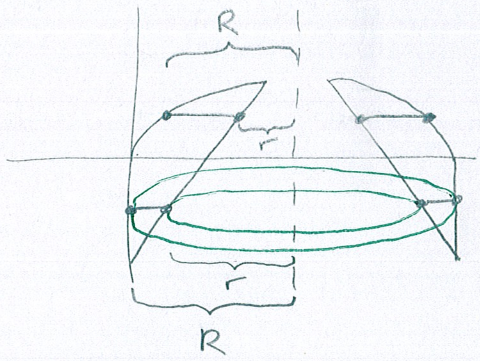
$0 = x^2-13x+36$

$0 = (x-4)(x-9)$

$x = 4, 9$

$y = x-6 = \cancel{-2}, 3$

[ii] using a dy integral **<u>(NOTE: You do NOT need to simplify your integrand.)</u>**



$$\boxed{\pi\underbrace{\int_0^3}_{1} \underbrace{((12-y^2)^2-(12-(y+6))^2)}_{2\frac{1}{2}}\,dy + \pi\underbrace{\int_{-6}^0}_{1} \underbrace{((12-0)^2-(12-(y+6))^2)}_{2}\,dy}$$

1

$y=\sqrt{x} \rightarrow x=y^2$

$y=x-6 \rightarrow x=y+6$

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$axis are semicircles.
Write, **<u>BUT DO NOT EVALUATE</u>**, an integral (or sum of integrals) for the volume of the solid.

$$\underbrace{\boxed{\frac{\pi}{8}}}_{1}\underbrace{\int_0^9}_{\frac{1}{2}} \underbrace{(\sqrt{x}-(x-6))^2}_{1}\,dx$$